

# Signal and Interference Leakage Minimization in MIMO Uplink-Downlink Cellular Networks

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**Abstract**—Linear processing in the spatial domain at the base stations (BSs) and at the users of MIMO cellular systems enables the control of both inter-cell and intra-cell interference. A number of iterative algorithms have been proposed that allow the BSs and the users to calculate the transmit-side and the receive-side linear processors in a distributed manner via message exchange based only on local channel state information. In this paper, a novel such strategy is proposed that requires the exchange of unitary matrices between BSs and users. Specifically, focusing on a general both uplink- and downlink-operated cells, the design of the linear processors is obtained as the alternating optimization solution of the problem of minimizing the weighted sum of the downlink and uplink inter-cell interference powers and of the signal power leaked in the space orthogonal to the receive subspaces. Intra-cell interference is handled via minimum mean square error (MMSE) or the zero-forcing (ZF) precoding for downlink-operated cells and via joint decoding for the uplink-operated cells. Numerical results validate the advantages of the proposed technique with respect to existing similar techniques that account only for the interference power in the optimization.

**Index Terms**—Linear precoding, interference alignment, uplink, downlink, MIMO cellular system.

## I. INTRODUCTION

Linear processing in the spatial domain at the base stations (BSs) and at the users of a Multi-Input Multi-Output (MIMO) cellular system is a well studied technique that enables the control of both inter-cell and intra-cell interference (see, e.g., [1]). A number of iterative algorithms have been proposed in the past few years for the design of the linear processors that are either centralized, see, e.g., [2] and references therein, or can be instead implemented in a decentralized way [1][3]–[7]. In the latter case, the BSs and the users calculate the transmit-side and the receive-side linear processors in a distributed manner via message exchange based only on local channel state information.

The distributed techniques in [1][3]–[7] differ in the information that is exchanged between the BSs and users and in the processing that is carried out at the two sides. Another key classification of these techniques can be done with respect to methods that apply to MIMO interference channels, i.e., cellular systems with a single user per cell, and techniques are suitable for to more general cellular systems with multiple users per cell. The interference leakage minimization (ILM) techniques of [3][4] require the exchange of unitary matrices

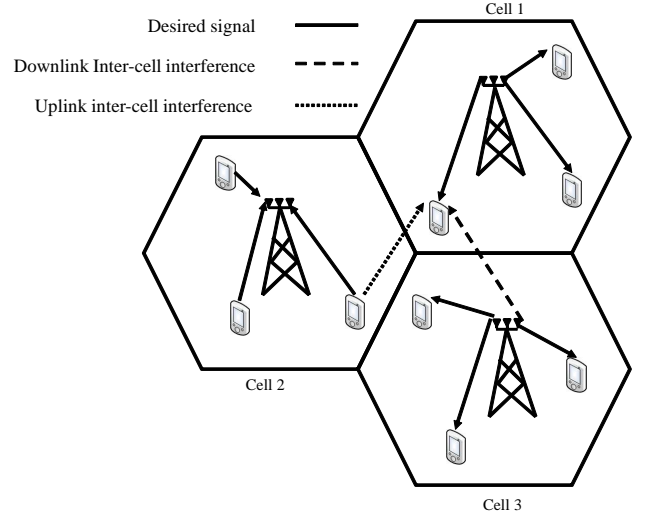


Figure 1. Multi-cell uplink-downlink MIMO system. Downlink and uplink inter-cell interference signal paths are shown for a user in cell 1 as an example.

between the two sides<sup>1</sup> and was proposed for a MIMO interference channel. References [6][7] generalize the ILM technique to a cellular system with an arbitrary number of users per cell, where the cells operate in either uplink or downlink. In contrast, the technique proposed in [1] requires the exchange of additional information beside unitary matrices and applies to the downlink of a general MIMO cellular system. In this regard, we observe that the transmission of unitary matrices is facilitated by the advances in the quantization over the Grassmann manifold (see, e.g., [9]) and is hence desirable, making the ILM scheme of [3][4][6][7] potentially more viable for practical implementation. The signal plus interference leakage minimization technique (SILM) of [5] modifies the ILM strategy by including in the cost function, not only the interference power, but also the power of the signal that is wasted in the space orthogonal to the receive subspaces. This scheme also requires the exchange of unitary matrices and was studied in [5] for MIMO interference channels.

In this paper, a novel iterative strategy is proposed that generalizes SILM [5] to a MIMO cellular system with an

<sup>1</sup>A different implementation based on pilot symbols and estimation is also possible, see [8].

arbitrary number of users per cell and in which each cell may be operated either in the uplink or in the downlink. Note that this mixed uplink-downlink configuration is known to be potentially advantageous, even in terms of degrees of freedom [11]. Specifically, following [6][7][10], the precoding matrix at the downlink-operated BSs is factorized into a unitary pre-processing matrix that handles inter-cell interference and a post-processing matrix that deals with intra-cell interference. The design of precoding and receiver-side matrices is obtained as the alternating optimization solution of the problem of minimizing the weighted sum of the inter-cell interference powers and of the signal power leaked in the space orthogonal to the receive subspaces. The post-processing precoding matrices at the downlink BSs are then calculated using the minimum mean square error (MMSE) or the zero-forcing (ZF) criteria, where the latter was considered in [6]. Numerical results validate the advantages of the proposed technique with respect to the ILM strategy of [6][7].

The rest of the paper is organized as follows. Sec. II presents the system model. Sec. III formulates the problem and introduces the proposed algorithm. Evaluation of the performance of the proposed algorithm is presented in Sec. IV via numerical results. Finally, we conclude with some remarks in Sec. V.

*Notation:* Bold uppercase letters denote matrices and bold lowercase letters denote column vectors. The notations  $\mathbb{E}$  and  $\mathbb{C}$  are the expectation operator and the complex field, respectively.  $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  represents the circularly symmetric distribution with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$ .  $\text{tr}(\mathbf{A})$  denotes the trace of the matrix  $\mathbf{A}$  and  $\mathbf{A}^H$  is the conjugate transpose of matrix  $\mathbf{A}$ .  $v_{\min}^b(\mathbf{A})$  and  $v_{\max}^b(\mathbf{A})$  are the truncated unitary matrices that consist of the  $b$  eigenvectors corresponding to the  $b$  smallest and  $b$  largest eigenvalues of the non-negative definite matrix  $\mathbf{A}$ , respectively. The Frobenius norm of a matrix  $\mathbf{A}$  is denoted as  $\|\mathbf{A}\|_F$ .  $\mathbf{I}$  represents the identity matrix.  $\mathbf{A}^\perp$  represents a unitary matrix that spans the subspace orthogonal to the column space of the unitary matrix  $\mathbf{A}$ .

## II. SYSTEM MODEL

We study the multi-cell MIMO system shown in Fig. 1, in which a subset of  $L_u$  cells operates in the uplink while the remaining  $L_d$  cells operate in the downlink. We will use the subscripts 'u' and 'd' throughout to denote the uplink and downlink cells, respectively. Each BS has  $N_b$  transmit/ receive antennas and each mobile user has  $N_m$  receive/ transmit antennas in the downlink/ uplink cells. There are  $K$  users per cell. We emphasize that it is straightforward to generalize the analysis to arbitrary numbers of users per cell and antennas.

We focus on a standard channel model with flat-fading MIMO channels that remain constant throughout the transmission block. Starting with the downlink cells, the signal  $\mathbf{y}_{\alpha_d k} \in \mathbb{C}^{N_m \times 1}$  received by the  $k$ th user in cell  $\alpha_d \in \{1, \dots, L_d\}$  is then given by

$$\begin{aligned} \mathbf{y}_{\alpha_d k} = & \underbrace{\mathbf{H}_{\alpha_d k}^{\alpha_d} \mathbf{x}_{\alpha_d k}}_{\text{desired signal}} + \underbrace{\sum_{j=1, j \neq k}^K \mathbf{H}_{\alpha_d k}^{\alpha_d} \mathbf{x}_{\alpha_d j}}_{\text{intra-cell interference}} \\ & + \underbrace{\sum_{\alpha_u=1}^{L_u} \sum_{j=1}^K \mathbf{H}_{\alpha_d k}^{\alpha_u j} \mathbf{x}_{\alpha_u j}}_{\text{uplink inter-cell interference}} \\ & + \underbrace{\sum_{\beta_d=1, \beta_d \neq \alpha_d}^{L_d} \sum_{j=1}^K \mathbf{H}_{\alpha_d k}^{\beta_d} \mathbf{x}_{\beta_d j}}_{\text{downlink inter-cell interference}} + \mathbf{n}_{\alpha_d k}, \quad (1) \end{aligned}$$

where  $\mathbf{H}_{\alpha_d k}^{\beta_d}$  is the  $N_m \times N_b$  channel matrix from BS  $\beta_d$  to user  $k$  in cell  $\alpha_d$ , where  $\alpha_d, \beta_d \in \{1, \dots, L_d\}$ ;  $\mathbf{x}_{\beta_d j} \in \mathbb{C}^{N_b \times 1}$  is the transmitted signal vector from BS  $\beta_d$  intended to user  $j$ ;  $\mathbf{H}_{\alpha_d k}^{\alpha_u j}$  is the  $N_m \times N_m$  channel matrix from user  $j$  in cell  $\alpha_u$  to user  $k$  in cell  $\alpha_d$  and  $\mathbf{n}_{\alpha_d k}$  denotes the thermal noise at the considered user, which is assumed to be distributed as  $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ . The first term on the right-hand side of the equality is the desired signal, the second term is the intra-cell interference, the third term is the uplink inter-cell interference from all the users in the  $L_u$  uplink cells and the forth term is the downlink inter-cell interference from all the BSs in the  $L_d$  downlink cells other than BS  $\alpha_d$ .

As for the uplink cells, the signal  $\mathbf{y}_{\alpha_u} \in \mathbb{C}^{N_b \times 1}$  received by BS  $\alpha_u \in \{1, \dots, L_u\}$  is given by

$$\begin{aligned} \mathbf{y}_{\alpha_u} = & \underbrace{\sum_{k=1}^K \mathbf{H}_{\alpha_u}^{\alpha_u k} \mathbf{x}_{\alpha_u k}}_{\text{desired signal}} + \underbrace{\sum_{\beta_u=1, \beta_u \neq \alpha_u}^{L_u} \sum_{k=1}^K \mathbf{H}_{\alpha_u}^{\beta_u k} \mathbf{x}_{\beta_u k}}_{\text{uplink inter-cell interference}} \\ & + \underbrace{\sum_{\alpha_d=1}^{L_d} \sum_{j=1}^K \mathbf{H}_{\alpha_u}^{\alpha_d} \mathbf{x}_{\alpha_d j}}_{\text{downlink inter-cell interference}} + \mathbf{n}_{\alpha_u}, \quad (2) \end{aligned}$$

where  $\mathbf{H}_{\alpha_u}^{\beta_u k}$  is the  $N_m \times N_b$  channel matrix from user  $k$  in cell  $\beta_u$  to BS  $\alpha_u$ , where  $\alpha_u, \beta_u \in \{1, \dots, L_u\}$ ;  $\mathbf{x}_{\beta_u k} \in \mathbb{C}^{N_m \times 1}$  is the transmitted signal vector from user  $k$  in cell  $\beta_u$ ;  $\mathbf{H}_{\alpha_u}^{\alpha_d}$  is the  $N_b \times N_b$  channel matrix from BS  $\alpha_d$  to BS  $\alpha_u$  and  $\mathbf{n}_{\alpha_u}$  denotes the thermal noise at the considered BS, which is assumed to be distributed as  $\mathcal{CN}(\mathbf{0}, \mathbf{I})$ . The first and second terms on the right-hand side of the equality are interpreted as in (1). The third term is the downlink inter-cell interference from all the BSs in the  $L_d$  downlink cells.

We assume that, in the downlink, we have the power constraint

$$\sum_{k=1}^K \mathbb{E} \left[ \|\mathbf{x}_{\alpha_d k}\|^2 \right] = P \quad (3)$$

for all  $\alpha_d \in \{1, \dots, L_d\}$ . In the uplink, equal power is used

by all users yielding

$$\mathbb{E} \left[ \|\mathbf{x}_{\alpha_d k}\|^2 \right] = P/K, \quad (4)$$

for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $k \in \{1, \dots, K\}$ .

In the downlink cells, for the transmission from BS  $\alpha_d$  to user  $k$ , BS  $\alpha_d$  chooses a unitary precoding matrix  $\mathbf{V}_{\alpha_d k} \in \mathbb{C}^{N_b \times s}$ , where  $s$  is the number of data streams per user. We assume throughout that the number of data streams does not exceed the number of receive antennas at each user, i.e.,  $s \leq N_m$ , and that the total number of data streams at each BS does not exceed the number of transmit antennas, i.e.,  $Ks \leq N_b$ . Moreover, for each user  $k$  in the downlink cell  $\alpha_d$ , a unitary matrix  $\mathbf{G}_{\alpha_d k} \in \mathbb{C}^{N_m \times (N_m - s)}$  is selected that defines its *interference subspace* as in [5]. This is in the sense that the user pre-processes the received signal as  $(\mathbf{G}_{\alpha_d k}^\perp)^H \mathbf{y}_{\alpha_d k}$ , hence filtering out the received signal component that lie within the interference subspace. We refer to the column space spanned by  $\mathbf{G}_{\alpha_d k}^\perp$  as the *receive subspace* for user  $k$  in cell  $\alpha_d$ .

In the uplink cells, for the transmission from user  $k$  to BS  $\alpha_u$ , user  $k$  chooses a precoding matrix  $\mathbf{G}_{\alpha_u k}^\perp \in \mathbb{C}^{N_m \times s}$ , where  $s$  is the number of data streams sent by each user. As in the downlink case, we assume that the number of data streams satisfies  $s \leq N_m$  and  $Ks \leq N_b$ . For each BS  $\alpha_u$ , a unitary matrix  $\mathbf{V}_{\alpha_u} \in \mathbb{C}^{N_m \times Ks}$  is used as the *receiving subspace*. We observe that the notation for the uplink cells is selected so as to be consistent with that of the downlink cells.

In the next section, we will discuss how to design downlink matrices  $\mathbf{V}_{\alpha_d k} \in \mathbb{C}^{N_b \times s}$  and  $\mathbf{G}_{\alpha_d k} \in \mathbb{C}^{N_m \times (N_m - s)}$  for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $k \in \{1, \dots, K\}$  and the uplink matrices  $\mathbf{V}_{\alpha_u} \in \mathbb{C}^{N_m \times Ks}$  and  $\mathbf{G}_{\alpha_u k} \in \mathbb{C}^{N_m \times (N_m - s)}$  for all  $\alpha_u \in \{1, \dots, L_u\}$  and  $k \in \{1, \dots, K\}$ .

### III. SIGNAL AND INTERFERENCE LEAKAGE MINIMIZATION

Reference [5] proposed the SILM scheme for the design of the precoding and receiving matrices for the special case  $K = 1$ , i.e., for a MIMO interference channel. Note that, in this case, we can set  $L_u = 0$  or  $L_d = 0$  with no loss of generality. The SILM scheme aims at striking a balance between two objectives: 1) minimizing the interference power received by the users and BSs in the corresponding receiving subspaces; 2) minimizing the signal power that is wasted in the corresponding interference subspaces. This is done by adopting as the optimization criterion the weighted sum of the power of the interference leaked in the receive subspace and the power of the signal wasted in the interference subspace. Specifically, an alternating optimization method is proposed in which the precoding and receiving matrices are optimized iteratively until convergence to a local minimum of the performance criterion.

The proposed SILM scheme for the multicell uplink-downlink MIMO scenario at hand combines the idea of SILM with the two-step precoding approach for downlink channels studied in [6][7][10]. Specifically, the overall precoding matrix

$\mathbf{V}_{\alpha_d} = [\mathbf{V}_{\alpha_d 1}, \mathbf{V}_{\alpha_d 2}, \dots, \mathbf{V}_{\alpha_d K}]$  used at the BS  $\alpha_d$  is written as the product

$$\mathbf{V}_{\alpha_d} = \mathbf{V}'_{\alpha_d} \mathbf{V}''_{\alpha_d}, \quad (5)$$

where  $\mathbf{V}'_{\alpha_d} = [\mathbf{V}'_{\alpha_d 1}, \dots, \mathbf{V}'_{\alpha_d K}] \in \mathbb{C}^{N_b \times Ks}$ , with  $\mathbf{V}'_{\alpha_d k} \in \mathbb{C}^{N_b \times s}$ , is a unitary matrix that is designed to handle *up-link and downlink inter-cell interference*, while  $\mathbf{V}''_{\alpha_d} = [\mathbf{V}''_{\alpha_d 1}, \dots, \mathbf{V}''_{\alpha_d K}] \in \mathbb{C}^{Ks \times Ks}$ , with  $\mathbf{V}''_{\alpha_d k} \in \mathbb{C}^{Ks \times s}$ , is used to mitigate *intra-cell interference*. In the following, we discuss the design of the unitary downlink matrices  $\mathbf{V}'_{\alpha_d}$  and  $\mathbf{G}_{\alpha_d k}$ , for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $k \in \{1, \dots, K\}$  along with the unitary uplink matrices  $\mathbf{V}'_{\alpha_u}$  and  $\mathbf{G}_{\alpha_u k}$ , for all  $\alpha_u \in \{1, \dots, L_u\}$  and  $k \in \{1, \dots, K\}$ , with the aim of handling inter-cell interference. We then detail the calculation of the intra-cell precoding matrices  $\mathbf{V}''_{\alpha_d}$ .

#### A. Uplink-Downlink Inter-Cell Precoding/Equalization

In order to design the precoding and decoding matrices mentioned above, we propose to minimize the sum

$$\sum_{\alpha_d=1}^{L_d} \sum_{k=1}^K I_{\alpha_d k} + \sum_{\alpha_u=1}^{L_u} \sum_{k=1}^K I_{\alpha_u k}, \quad (6)$$

where  $I_{\alpha_d k}$  is in turn defined as the weighted sum

$$\begin{aligned} I_{\alpha_d k} = & \sum_{\beta_d=1, \beta_d \neq \alpha_d}^{L_d} \left\| (\mathbf{G}_{\alpha_d k}^\perp)^H \mathbf{H}_{\alpha_d k}^{\beta_d} \mathbf{V}'_{\beta_d} \right\|_F^2 \\ & + \sum_{\alpha_u=1}^{L_u} \sum_{j=1}^K \left\| (\mathbf{G}_{\alpha_d k}^\perp)^H \mathbf{H}_{\alpha_d k}^{\alpha_u j} \mathbf{G}_{\alpha_u j}^\perp \right\|_F^2 \\ & + w \left\| \mathbf{G}_{\alpha_d k}^H \mathbf{H}_{\alpha_d k}^{\alpha_d} \mathbf{V}'_{\alpha_d} \right\|_F^2, \end{aligned} \quad (7)$$

with  $w \geq 0$  being a given weight, and  $I_{\alpha_u k}$  is defined as the weighted sum

$$\begin{aligned} I_{\alpha_u k} = & \sum_{\beta_u=1, \beta_u \neq \alpha_u}^{L_u} \left\| \mathbf{V}_{\alpha_u}^H \mathbf{H}_{\alpha_u}^{\beta_u k} \mathbf{G}_{\beta_u k}^\perp \right\|_F^2 \\ & + \sum_{\alpha_d=1}^{L_d} \sum_{j=1}^K \left\| \mathbf{V}_{\alpha_u}^H \mathbf{H}_{\alpha_u}^{\alpha_d} \mathbf{V}'_{\alpha_d} \right\|_F^2 \\ & + w \left\| \mathbf{V}_{\alpha_u}^H \mathbf{H}_{\alpha_u}^{\alpha_u k} \mathbf{G}_{\alpha_u k}^\perp \right\|_F^2. \end{aligned} \quad (8)$$

The expression (7) is the sum in order of appearance, of the inter-cell downlink and uplink interference powers (assuming  $\mathbf{V}''_{\alpha_d} = (P/K)\mathbf{I}$ ) and of the signal power wasted in the interference subspace for user  $k$  in cell  $\alpha_u$ , where the latter term is weighted by  $w$ , and (8) has a similar interpretation.

The optimization of (7) is performed by alternating between the minimization over the receive-side matrices  $\mathbf{G}_{\alpha_d k}$  and  $\mathbf{V}_{\alpha_u}$  for fixed transmit-side matrices  $\mathbf{V}'_{\alpha_d}$  and  $\mathbf{G}_{\alpha_u j}$  and the minimization over the transmit-side matrices  $\mathbf{G}_{\alpha_u j}$  and  $\mathbf{V}'_{\alpha_d}$  for fixed receive-side matrices  $\mathbf{G}_{\alpha_d k}$  and  $\mathbf{V}_{\alpha_u}$  following the procedure described in Table Algorithm 1. Specifically, in Step 2, the decoding matrices  $\mathbf{G}_{\alpha_d k}^\perp$  and  $\mathbf{V}_{\alpha_u}$  are obtained as

$$\mathbf{G}_{\alpha_d k} = v_{max}^{(N_m - s)} \left( \vec{\mathbf{Q}}_{\alpha_d k} \right), \quad (9)$$

$$\mathbf{V}_{\alpha_u} = v_{min}^{Ks} \left( \overleftarrow{\mathbf{Q}}_{\alpha_u} \right), \quad (10)$$

where

$$\begin{aligned} \vec{\mathbf{Q}}_{\alpha_d k} = & \sum_{\beta_d=1, \beta_d \neq \alpha_d}^{L_d} \mathbf{H}_{\alpha_d k}^{\beta_d} \mathbf{V}_{\beta_d}' \mathbf{V}_{\beta_d}'^H \mathbf{H}_{\alpha_d k}^{\beta_d H} \\ & + \sum_{\alpha_u=1}^{L_u} \sum_{j=1}^K \mathbf{H}_{\alpha_d k}^{\alpha_u j} \mathbf{G}_{\alpha_u j}^{\perp} (\mathbf{G}_{\alpha_u j}^{\perp})^H \mathbf{H}_{\alpha_d k}^{\alpha_u j H} \\ & - w \mathbf{H}_{\alpha_d k}^{\alpha_d} \mathbf{V}_{\alpha_d}' \mathbf{V}_{\alpha_d}'^H \mathbf{H}_{\alpha_d k}^{\alpha_d H} \end{aligned} \quad (11)$$

and

$$\begin{aligned} \overleftarrow{\mathbf{Q}}_{\alpha_u} = & \sum_{\beta_u=1, \beta_u \neq \alpha_u}^{L_u} \sum_{j=1}^K \mathbf{H}_{\alpha_u}^{\beta_u j H} \mathbf{G}_{\beta_u j} \mathbf{G}_{\beta_u j}^H \mathbf{H}_{\alpha_u}^{\beta_u j} \\ & + \sum_{\beta_u=1}^{L_u} \mathbf{H}_{\alpha_u}^{\beta_u H} \mathbf{V}_{\beta_u} \mathbf{V}_{\beta_u}^H \mathbf{H}_{\alpha_u}^{\beta_u} \\ & - w \sum_{j=1}^K \mathbf{H}_{\alpha_u}^{\alpha_u j H} \mathbf{G}_{\alpha_u j} \mathbf{G}_{\alpha_u j}^H \mathbf{H}_{\alpha_u}^{\alpha_u j}. \end{aligned} \quad (12)$$

The first term on the right-hand side of (11) is the covariance matrix of the downlink inter-cell interference at user  $k$  in cell  $\alpha_d$ ; the second term is the covariance matrix of the uplink inter-cell interference at user  $k$  in cell  $\alpha_d$ ; and the third term is the weighted covariance matrix of the desired signal for all the users in cell  $\alpha_d$  as observed by user  $k$  in cell  $\alpha_d$ . The covariance matrix (12) has a similar interpretation.

In Step 3, the precoding matrices  $\mathbf{V}_{\alpha_d}'$  and  $\mathbf{G}_{\alpha_u k}^{\perp}$  are similarly obtained as

$$\mathbf{V}_{\alpha_d}' = v_{min}^{Ks} \left( \vec{\mathbf{Q}}_{\alpha_d} \right), \quad (13)$$

$$\mathbf{G}_{\alpha_u k} = v_{max}^{(N_m - s)} \left( \overleftarrow{\mathbf{Q}}_{\alpha_u k} \right), \quad (14)$$

where

$$\begin{aligned} \vec{\mathbf{Q}}_{\alpha_d} = & \sum_{\beta_d=1, \beta_d \neq \alpha_d}^{L_d} \sum_{j=1}^K \mathbf{H}_{\beta_d j}^{\alpha_d H} \mathbf{G}_{\beta_d j} \mathbf{G}_{\beta_d j}^H \mathbf{H}_{\beta_d j}^{\alpha_d} \\ & + \sum_{\alpha_u=1}^{L_u} \sum_{j=1}^K \mathbf{H}_{\alpha_u j}^{\alpha_d H} \mathbf{G}_{\alpha_u j} \mathbf{G}_{\alpha_u j}^H \mathbf{H}_{\alpha_u j}^{\alpha_d} \\ & - w \sum_{j=1}^K \mathbf{H}_{\alpha_d j}^{\alpha_d H} \mathbf{G}_{\alpha_d j} \mathbf{G}_{\alpha_d j}^H \mathbf{H}_{\alpha_d j}^{\alpha_d} \end{aligned} \quad (15)$$

and

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**Algorithm 1** Signal and Interference Leakage Minimization (SILM) for the multicell MIMO downlink

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**Step 1:** Start with arbitrary unitary precoding matrices  $\mathbf{V}_{\alpha_d}'$  for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $\mathbf{G}_{\alpha_u k}$  for all  $\alpha_u \in \{1, \dots, L_u\}$ .  
**Step 2:** Compute  $\mathbf{G}_{\alpha_d k}$  as in (9) for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $k \in \{1, \dots, K\}$  and  $\mathbf{V}_{\alpha_u}$  as in (10) for all  $\alpha_u \in \{1, \dots, L_u\}$ .  
**Step 3:** Compute  $\mathbf{V}_{\alpha_d}'$  as in (13) for all  $\alpha_d \in \{1, \dots, L_d\}$  and  $\mathbf{G}_{\alpha_u k}$  as in (14) for all  $\alpha_u \in \{1, \dots, L_u\}$  and  $k \in \{1, \dots, K\}$ .

**Step 4:** If a convergence criterion is satisfied, go to Step 5; otherwise go back to Step 2.

**Step 5:** Compute  $\mathbf{V}_{\alpha_d}''$  using (18).

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$$\begin{aligned} \overleftarrow{\mathbf{Q}}_{\alpha_u k} = & \sum_{\alpha_d=1}^{L_d} \sum_{j=1}^K \mathbf{H}_{\alpha_d j}^{\alpha_u k H} \mathbf{G}_{\alpha_d j} \mathbf{G}_{\alpha_d j}^H \mathbf{H}_{\alpha_d j}^{\alpha_u k} \\ & + \sum_{\beta_u=1, \beta_u \neq \alpha_u}^{L_u} \sum_{j=1}^K \mathbf{H}_{\beta_u}^{\alpha_u k H} \mathbf{V}_{\beta_u} \mathbf{V}_{\beta_u}^H \mathbf{H}_{\beta_u}^{\alpha_u k} \\ & - w \sum_{j=1}^K \mathbf{H}_{\alpha_u}^{\alpha_u k H} \mathbf{V}_{\alpha_u} \mathbf{V}_{\alpha_u}^H \mathbf{H}_{\alpha_u}^{\alpha_u k}. \end{aligned} \quad (16)$$

The first term on the right-hand side of (15) is the covariance matrix of the downlink inter-cell interference caused by BS  $\alpha_d$  to all downlink users; the second term is the covariance matrix of the downlink inter-cell interference caused by BS  $\alpha_d$  to all the uplink BSs; and the third term is weighted covariance matrix of the desired signal to all the users in cell  $\alpha_d$  that is leaked in the interference subspaces. The covariance matrix (16) has a similar interpretation.

**Remark 1:** The alternating optimizations algorithm in Table Algorithm 1 can be implemented in a distributed fashion where Step 2 is carried out in parallel by all the downlink users and uplink BSs, while Step 3 is performed in parallel by all the downlink BSs and uplink users. In both steps, only local channel state information is needed if the users and BSs, e.g., BS  $\alpha_d$  only needs to know the channels  $\mathbf{H}_{\beta_d k}^{\alpha_d}$  for all  $\beta_d \in \{1, \dots, L_d\}$  and  $k \in \{1, \dots, K\}$ . Moreover, BSs and users need to exchange unitary matrices during the operation of the algorithm in order to calculate the covariance matrices (11), (12), (15) and (16).  $\square$

Given the matrices  $\mathbf{G}_{\alpha_d k}$  and  $\mathbf{V}_{\alpha_d}'$ , the effective channel observed by the  $K$  users in the downlink cell  $\alpha_d$  from BS  $\alpha_d$  is given as

$$\tilde{\mathbf{H}}_{\alpha_d} = \left[ (\mathbf{G}_{\alpha_d k}^{\perp})^H \mathbf{H}_{\alpha_d 1}^{\alpha_d} \mathbf{V}_{\alpha_d}'; \dots; (\mathbf{G}_{\alpha_d k}^{\perp})^H \mathbf{H}_{\alpha_d K}^{\alpha_d} \mathbf{V}_{\alpha_d}' \right]. \quad (17)$$

Each BS  $\alpha_d$  then precodes over this channel so as to control intra-cell interference. Here, we adopt a linear MMSE intra-cell precoder, which is given as

$$\mathbf{V}_{\alpha_d}'' = \left( \mu_{\alpha_d} \mathbf{I} + \tilde{\mathbf{H}}_{\alpha_d} \tilde{\mathbf{H}}_{\alpha_d}^H \right)^{-1} \tilde{\mathbf{H}}_{\alpha_d}^H, \quad (18)$$

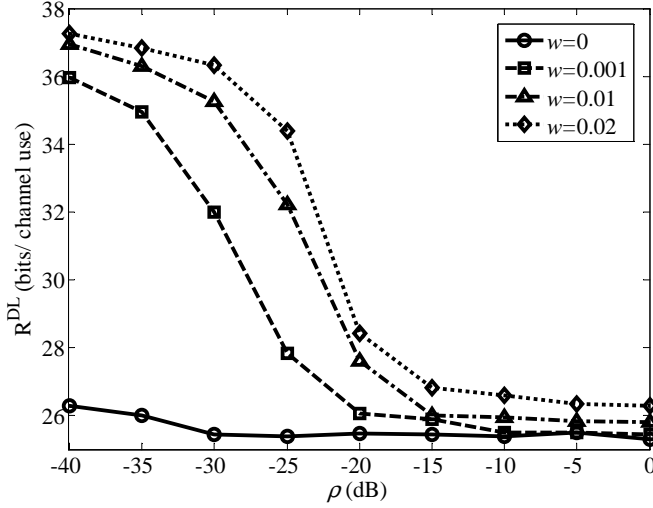


Figure 2. Sum-rate  $R^{DL}$  for the downlink with MMSE precoding versus the inter-cell interference gain  $\rho$  ( $L_d = 4, L_u = 0, K = 4, N_b = N_m = 5, s = 1, SNR = 10dB$ ).

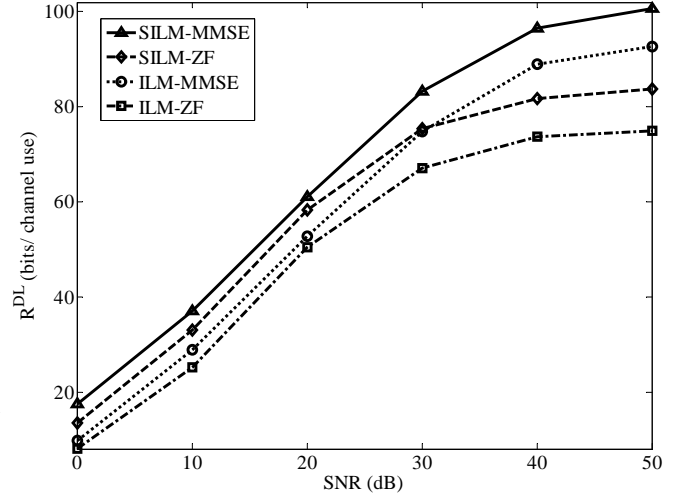


Figure 3. Sum-rate  $R^{DL}$  for the downlink channel versus SNR for ILM-ZF, SILM-ZF, ILM-MMSE and SILM-MMSE ( $L_d = 4, L_u = 0, K = 5, N_b = N_m = 5, s = 1, \rho = -20dB$ ).

where  $\mu_{\alpha_d}$  is a scalar that must be selected such that  $\text{tr}[\mathbf{V}_{\alpha_d} \mathbf{V}_{\alpha_d}^H] = P$ . Note that with  $\mu_{\alpha_d} = 0$ , the MMSE solution in (18) reduces to the ZF design considered in [6].

#### B. Sum-rate

Given the designed precoding and decoding matrices, assuming that all interference is treated as noise, the sum-rate of the downlink cells can be computed as in (19) and the the sum-rate of the uplink cells can be obtained in a similar fashion as in (20). Note that the latter assumes equal power allocation per stream and joint decoding of all the uplink users in a cell.

### IV. NUMERICAL RESULTS

In this section, we present some numerical results for the schemes under study. We assume that all the channel matrices corresponding to a BS and a user in the same cell are independent and identically distributed (i.i.d.) as  $\mathcal{CN}(0, 1)$  and all channel matrices corresponding to a BS and a user in a different cell are i.i.d. as  $\mathcal{CN}(0, \rho^2)$ , where  $\rho$  can be interpreted as the inter-cell interference gain. We define the signal-to-noise ratio (SNR) as being equal to  $P$ .

We first consider the special case where all cells operate in the downlink. Fig. 2 plots the downlink sum-rate  $R^{DL}$  for the downlink versus the inter-cell interference gain  $\rho$  for  $L_d = 4, L_u = 0, K = 4, N_b = N_m = 5, s = 1$  and  $SNR = 10dB$ . The performance of SILM, which corresponds to  $w > 0$ , is shown for MMSE precoding. As  $\rho$  decreases, the performance of ILM, which corresponds to  $w = 0$ , is significantly degraded as compared to SILM since the contribution of the inter-cell interference becomes relatively less relevant. It is also noted that values of  $w$  larger than 0.02 do not further improve the performance (not shown).

Fig. 3 plots the downlink sum-rate  $R^{DL}$  for the downlink versus the SNR for  $L_d = 4, L_u = 0, K = 5, N_b = N_m =$

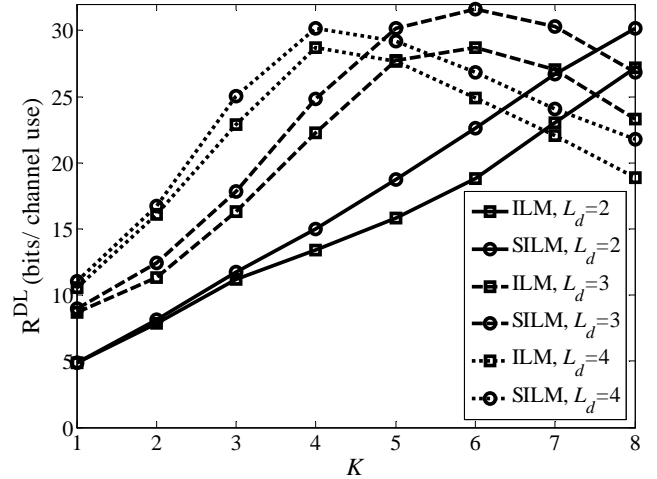


Figure 4. Sum-rate  $R^{DL}$  for the downlink versus  $K$  for ILM and SILM ( $L_d = 2, 3, 4, L_u = 0, N_b = N_m = 5, s = 1, \rho = -20dB$ ).

$5, s = 1$  and  $\rho = -20dB$ . Following [5], the weights  $w$  corresponding to the SNR values  $[0 \ 10 \ 20 \ 30 \ 40 \ 50]$  are selected as  $[0.02 \ 0.02 \ 0.005 \ 0.003 \ 0.002 \ 0.001]$ . Note that the weight value decreases with the SNR, reflecting the enhanced role of interference in the high-SNR regime. The performance of SILM with MMSE and ZF intra-cell precoding is compared to the ILM scheme. It is confirmed that SILM-based algorithms outperform ILM schemes. Moreover, MMSE precoding significantly improves the sum-rate over ZF.

Fig. 4 plots the sum-rate  $R^{DL}$  versus the number  $K$  of users per cell for ILM and SILM for  $L_d = 2, 3, 4, L_u = 0, N_b = N_m = 5, s = 1$  and  $\rho = -20dB$ . It is seen that the gain of SILM over ILM decreases as the number of cells and/or of users per cell increases, and hence as the performance of the system becomes increasingly interference limited. Note

$$\begin{aligned}
R^{DL} = & \sum_{\alpha_d=1}^{L_d} \sum_{k=1}^K \log \left| \mathbf{I} + \left( \mathbf{I} + (\mathbf{G}_{\alpha_d k}^\perp)^H \right. \right. \\
& \left( \sum_{i=1, i \neq k}^K \mathbf{H}_{\alpha_d k}^{\alpha_d H} \mathbf{V}_{\alpha_d i} \mathbf{V}_{\alpha_d i}^H \mathbf{H}_{\alpha_d k}^\alpha + \sum_{\beta_d=1, \beta_d \neq \alpha_d}^{L_d} \mathbf{H}_{\alpha_d k}^{\beta_d} \mathbf{V}_{\beta_d} \mathbf{V}_{\beta_d}^H \mathbf{H}_{\alpha_d k}^{xH} \right. \\
& \left. \left. + \frac{P}{Kd} \sum_{\alpha_u=1}^{L_u} \sum_{j=1}^K \mathbf{H}_{\alpha_d k}^{\alpha_u j} \mathbf{G}_{\alpha_u j}^\perp (\mathbf{G}_{\alpha_u j}^\perp)^H \mathbf{H}_{\alpha_d k}^{\alpha_u j H} \right) \mathbf{G}_{\alpha_d k}^\perp \right)^{-1} (\mathbf{G}_{\alpha_d k}^\perp)^H \mathbf{H}_{\alpha_d k}^{\alpha_d} \mathbf{V}_{\alpha_d k} \mathbf{V}_{\alpha_d k}^H \mathbf{H}_{\alpha_d k}^{\alpha_d H} \mathbf{G}_{\alpha_d k}^\perp \left. \right|. \quad (19)
\end{aligned}$$

$$\begin{aligned}
R^{UL} = & \sum_{\alpha_u=1}^{L_u} \log \left| \mathbf{I} + \frac{P}{Kd} (\mathbf{I} + \mathbf{V}_{\alpha_u}^H \right. \\
& \left( \frac{P}{Kd} \sum_{\beta_u=1, \beta_u \neq \alpha_u}^{L_u} \sum_{i=1}^K (\mathbf{H}_{\beta_u i}^{\alpha_u k})^H \mathbf{G}_{\beta_u i}^\perp (\mathbf{G}_{\beta_u i}^\perp)^H \mathbf{H}_{\beta_u i}^{\alpha_u k} + \sum_{\alpha_d=1}^{L_d} \sum_{j=1}^K \mathbf{H}_{\alpha_u k}^{\alpha_d j H} \mathbf{V}_{\alpha_d j} \mathbf{V}_{\alpha_d j}^H \mathbf{H}_{\alpha_u k}^{\alpha_d j} \right) \mathbf{V}_{\alpha_u} \left. \right)^{-1} \\
& \left. \mathbf{V}_{\alpha_u}^H \sum_{k=1}^K (\mathbf{H}_{\alpha_u k}^{\alpha_u})^H \mathbf{G}_{\alpha_u k}^\perp (\mathbf{G}_{\alpha_u k}^\perp)^H \mathbf{H}_{\alpha_u k}^{\alpha_u} \mathbf{V}_{\alpha_u} \right|. \quad (20)
\end{aligned}$$

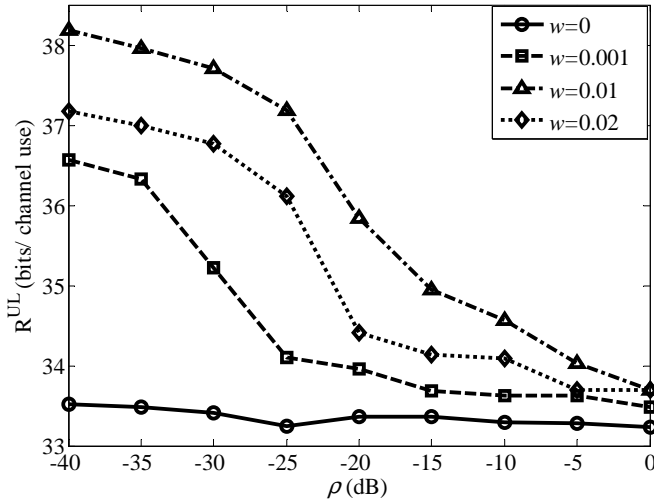


Figure 5. Sum-rate  $R^{UL}$  for the uplink versus  $\rho$  ( $L_d = 0, L_u = 4, K = 4, N_b = N_m = 5, s = 1, SNR = 10dB$ ).

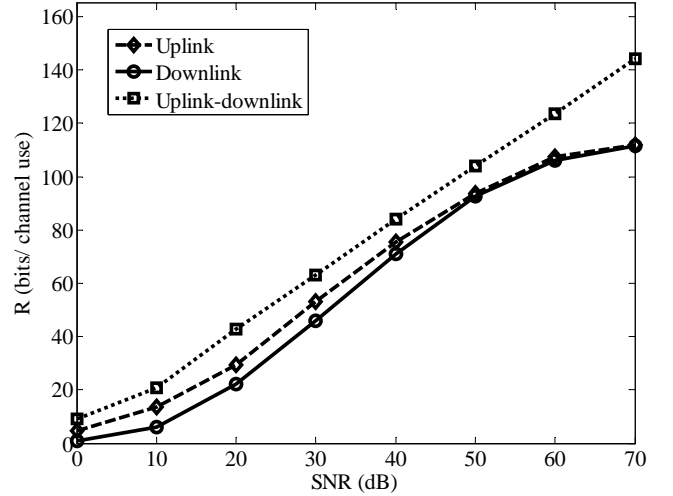


Figure 6. Sum-rate  $R$  for the uplink-downlink versus SNR for SILM-MMSE ( $L_d = 2, L_u = 2, K = 2, N_b = N_m = 4, s = 1, \rho = -20dB$ ).

also that there is an optimal number of users  $K$  due to the assumption of fixed power allocation.

We now consider the case of all uplink cells. Fig. 5 plots the sum-rate  $R^{UL}$  for the uplink versus the inter-cell interference gain  $\rho$  for  $L_d = 0, L_u = 4, K = 4, N_b = N_m = 5, s = 1$  and  $SNR = 10dB$ . As observed for the downlink in Fig. 2, SILM becomes more advantageous as  $\rho$  decreases, and hence as the performance becomes less limited by the inter-cell interference as implicitly assumed by ILM. It is also noted that  $w = 0.02$  yields lower performance than  $w = 0.01$ .

Finally, Fig. 6 plots the sum-rate versus the SNR for a

system with four cells. We compare the performance of the downlink configuration with  $L_d = 4$  and  $L_u = 0$ , of the uplink with  $L_d = 0$  and  $L_u = 4$  and of the uplink-downlink configuration with  $L_d = 2$  and  $L_u = 2, K = 2, N_b = N_m = 4, s = 1$  and  $\rho = -20dB$ . The uplink configuration outperforms the downlink solution due to the assumed joint decoding of the intra-cell users that contrasts with the assumed linear intra-cell downlink precoding. It is also observed that the uplink-downlink configuration provides significant performance gains especially at high SNR, confirming the results in [11].

## V. CONCLUDING REMARKS

In this paper, a novel algorithm for the design of linear transmit- and receive-side processing has been proposed for a MIMO multicell system with different cells operating in either uplink and downlink. The algorithm is based on the minimization of the weighted sum of the interference power that is leaked outside the interference subspace and of the signal power that falls into the interference subspace as in [5]. The proposed technique generalizes the interference-leakage based approach of [5]-[6] and is shown via numerical results to have significant sum-rate gains over existing techniques.

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